Appendix A
An Overview of Probabilistic Analysis for Geotechnical Engineering Problems

Table of Contents

Purpose and Scope

This appendix provides an overview of the application of probabilistic methods to geotechnical engineering problems of interest to the Corps of Engineers, with emphasis on methodology suitable for assessing the comparative reliability of dams, levees, and other hydraulic structures in the context of planning studies. A number of probabilistic methods that can and have been applied to the problems of interest are reviewed and discussed. These are drawn from Corps guidance, literature that led to Corps guidance, literature and methodology not yet in Corps guidance but considered state-of-the-art, case histories of past analyses by the Corps and by others for similar problems, and recent remarks made in state-of-the-art invited papers. The intent of this review is to introduce the reader to the diversity of methodology and issues that are encompassed in geotechnical probabilistic analysis, and their relationships to each other and Corps methodology, so that the relative accuracy, advantages, and limitations of Corps' methodology can be better understood in this context.

Probabilistic Methods

Background. As used herein, the term *probabilistic methods* refers to a collection of techniques that may be called or include *reliability analysis, risk analysis, risk-based analysis, life-data analysis,* and other similar terms. Such techniques have been under development and have seen increasing application to engineering problems for 50 years, starting with Frudenthal (1947). Since that time, and increasingly in the last 20 years, a significant body of literature has been published, proposing and detailing various methodologies and applications. Application to structural engineering problems, especially as the basis of design codes (e.g., Ellingwood et al. 1980), has generally preceded applications in geotechnical engineering. Geotechnical problems often involve certain complexities not found in structural problems.

Background of Corps' Applications. As the Corps' workload shifted from the design of new structures to the rehabilitation of existing structures, it became necessary to develop rational methodology to compare alternative plans for rehabilitation of Corps' projects and prioritize expenditures for such work. The previous approach of seeking funds on the basis that a structure does not meet current criteria is unworkable when funds are insufficient for all desired rehabilitation projects (U.S. Army Corps of Engineers 1992). The resulting approach has been to apply risk analysis techniques. In such a risk analysis,

- Unsatisfactory performance events are identified and the probabilities of their occurrence over some time frame are estimated.

- Consequences of the unsatisfactory performance events are estimated.

- Changes in probability and consequences associated with alternative plans of improvement are estimated.

- Decisions are made based on the quantified risk and costs and benefits of reducing the risk.

Since 1992, the Corps has used probabilistic methods to evaluate engineering reliability in the planning process for major rehabilitation projects. The methodology used by the Corps has been selectively adapted from previously published work (e.g. Moses and Verma 1987; Wolff and Wang 1992; Shannon and Wilson, Inc., and Wolff 1994; Wolff et al. 1995) and a limited amount of guidance has been published (e.g., U.S. Army Corps of Engineers 1992, 1993, 1995a, 1995b). Methodology is under development for planning studies for levee projects, and is under consideration for dam safety evaluation. Nevertheless, the application of probabilistic methods is an evolving technology. As the Corps' experience base expands and new and unique problems are considered, it will continue to be necessary to identify suitable methodology, either drawn from outside sources or developed within the Corps, ahead of its publication as Corps guidance.

Framework. To account for various modes of performance and estimate the required probabilities of unsatisfactory performance within a time frame, engineers and planners develop an *event tree* and engineers estimate probability values for a number of events and conditional events leading to various performance states of the structure or component. (Event trees are further discussed in the next section). Event trees are a convenient pictorial method to represent complex networks of conditional probability problems. They are not in themselves related to any single probabilistic method. The required probability values could be estimated using one or more of three approaches:

a. Calculating the probability of unsatisfactory performance as **a function of uncertainty in parameter values** and in the analytical models, typically using first-order second-moment methods or simulation (Monte Carlo) methods.

b. Calculating the probability of occurrence of various events from **time-based probability distributions** based on the study of historical records of similar events and fitting probability functions to these data.

c. Estimating the probability of event occurrence (either within a time increment or conditional on a preceding event) by a **systematic process of eliciting expert opinion** and developing a consensus regarding the required values.

This classification of three approaches is similar to that described by Vick and Stewart (1996). A broad treatment of probabilistic methods, including some or all of these approaches is contained in a number of general texts. Notable among these are Ang and Tang (1975, 1985), Benjamin and Cornell (1970), Hahn and Shapiro (1967), Harr (1987), and Lewis (1996). The following sections further describe event trees and the above three probabilistic approaches.

Risk Analysis

Event Trees. The framework for risk analysis in most Corps' planning studies is an *event tree*. An event tree is a pictorial representation of sequences of events that may lead to favorable or unfavorable outcomes. A simple example of part of an event tree is shown in Figure 1. Each node on the tree represents a situation where two or more mutually exclusive events may occur, given that events leading to the node have already occurred. For each branch from a node, a conditional probability of occurrence is assigned (conditioned on reaching the node via the preceding events). The set of conditional probability values emanating from each node must total to unity. In accordance with the

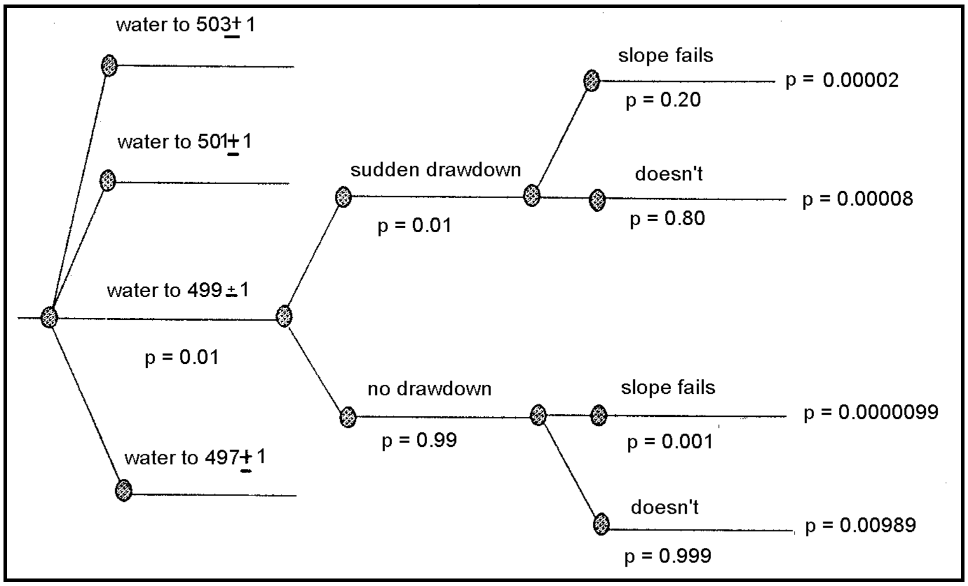

Figure 1. Partial event tree for slope stability given maximum water elevation in a time increment

total probability theorem, multiplying values along any path through the tree gives the probability of the outcome at the end of the path.

Example. The example in Figure 1 considers slope stability for a range of water elevations and illustrates how all three of the above-noted approaches may enter an event tree. Given a probability-of-annual-exceedance function for water level, a set of water levels can be discretized for analysis. For example, the probability that the maximum water level in a 1-year time increment is between elevation 498 and 500 can be taken as the difference in annual probabilities of exceedance for those elevations. For a slope stability, seepage, or other water-level-dependent analysis, the water level can be taken at the mid-point of the increment, i.e., 499. Probability of exceedance values for water levels are typically obtained using the second of the three approaches cited above, i.e., probability distributions are fit to historical data.

Given that the water level reaches elevation 499, there may or may not be a sudden drawdown event while the water level is at this elevation. The probability of this event might be estimated using frequency analysis of historical events. On the other hand, past events may be so sparse or dissimilar that probability can only be estimated by judgment (the third approach). Furthermore, the probability of drawdown may be a constant value per year, if its occurrence is totally random, or its value per year might be taken to increase with increasing water level if the likelihood of operational problems is considered to increase with water level.

Given a sudden drawdown event, the slope may or may not fail. As an analytical model and some understanding of the uncertainty in the model parameters are available for stability under a sudden drawdown condition, the conditional probability of failure given sudden drawdown can be estimated using first-order second-moment methods such as the Taylor's series method or the point estimate method. Note from the example in Figure 1 that the conditional probability of slope failure given sudden drawdown may be relatively high (0.20), but the preceding event of sudden drawdown might be quite low, leading to an overall low probability for the outcome of a sudden drawdown failure.

Time Basis of Reliability. Risk analyses for economic planning generally consider the risks in some defined time frame, typically 50 years. If the event-tree analysis is to determine the probability of unsatisfactory performance within some time increment, one of the underlying random variables must have a time-based definition, e.g. an annual probability of failure or an expected value of 0.xxxx failures per year. In the example shown, the time component is in the annual probability of exceedance function for water level. In the case of an electrical or mechanical part, the probability may have a time component related to time in service.

Hazard Functions. A *hazard function* gives the conditional probability of failure (or probability of event occurrence) per time increment given that no failure or event has occurred up to the considered time. Where time-based probability values are equal in each time increment, the hazard function has a constant value. This is referred to as a *Poisson process* and the lifetime is exponentially distributed. Floods and earthquakes are often assumed to be Poisson processes, with occurrence taken to be equally likely in any year. An increasing hazard function implies an increasing probability of failure or event occurrence as time elapses without such an event. An example of an increasing hazard function might be one for the formation of a window in a sheetpile cutoff due to corrosion, or the breakout of a seepage condition due to solutioning of limestone. An example of a decreasing hazard function might be one for the event of an undrained slope failure as pore pressures dissipate with time.

Fault Trees. An alternative to an event tree is a *fault tree*. Where an event tree starts with some initiating event (e.g. high water, or simply a year in the project life) and attempts to consider all subsequent possibilities, in a fault tree analysis one first identifies an outcome event of interest (e.g. loss of pool) and works backward to identify the necessary antecedent events. An advantage of fault tree analysis is that it may save time and be easier to accurately develop when specific and already-identified outcomes are of interest. For economic analysis of proposed rehabilitation projects, the event tree format has been preferred. However, if and as probabilistic methods are applied to dam safety issues, fault tree analysis may have some advantages. Vrouwenvelder (1987) uses fault tree analysis to assess the failure probability of Dutch levees.

Further References. Ang and Tang (1985) and Lewis (1966) both provide a number of detailed and illustrated examples of both event tree and fault tree analysis. Wu (1996) and Whitman (1996) also provide a brief treatment of event tree methodology in a geotechnical context.

Recent applications of event-tree analysis involving geotechnical problems at Corps projects include the Hodges Villages Dam rehabilitation report (U.S. Army Engineer Division, New England 1995) and the Walter F. George Dam rehabilitation report (U.S. Army Engineer District, Mobile 1997). In the Hodges Village Dam study, the initiating and time-related event is the occurrence of one of several maximum annual pool levels, each with some probability. Given each pool level, subsequent events are the occurrence of uncontrolled seepage leading to failure at one or more locations. The required conditional probabilities of seepage failure given pool level were developed using first-order, second-moment (FOSM) reliability methods in conjunction with finite-element seepage analyses.

In the Walter F. George study, the initiating and time-related event is the occurrence of excessive seepage in a solutioned limestone foundation. These are taken to be Weibull distributed with an increasing hazard function, which was fit to historical events at the site with some measure of judgment regarding the acceleration rate. Given such a seepage event, the event tree is filled out with conditional probabilities related to how well the seep is connected to pool and/or tailwater, and how likely or unlikely it is that the source of the seep will be detected and plugged before uncontrolled erosion occurs.

In both studies, the recommended remedial action was the construction of concrete cutoff walls in the foundation.

Random Variables

Random Variables and Distributions. The fundamental building blocks of probabilistic analyses are random variables. In mathematical terms, a *random variable* is a function defined on a sample space that assigns a probability or likelihood to each possible event within the sample space. In practical terms, a random variable is a variable for which the precise value is uncertain, but some probability can be assigned to its assuming any specific value (for discrete random variables) or being within any range of values (for continuous random variables).

Discrete random variables can only assume specific values. Some examples of discrete random variables encountered in geotechnical engineering include:

- Number of sand boils or seeps that may occur within length L in time period t.

- Number of levee overtoppings in length L in time period t.

- In general, the number of events in an increment of time or space.

Commonly employed models for discrete random variables include the binomial and Poisson distributions.

Continuous random variables can assume a continuous range of values over a domain, and probability values must be associated with some range within the domain. Some continuous random variables include:

- Undrained strength or cohesion of a clay stratum.

- Friction angle.

- Permeability.

- Exit gradient at the toe of a levee.

- Time to occurrence of an erosive seepage or scour event.

- Time to occurrence of any event.

Commonly employed models for continuous random variables include the normal, lognormal, and uniform distributions; however, there are a number of others, such as the beta distribution discussed by Harr (1987). Random variables are discussed in some detail (distributions, moments, etc.) in standard texts (Ang and Tang 1975, 1985; Benjamin and Cornell 1970; Hahn and Shapiro 1967: Harr 1987; Lewis 1996), Corps-sponsored research reports (Wolff and Wang 1992, Shannon and Wilson, Inc., and Wolff 1994; Wolff et al. 1995) and in Corps' guidance (U.S. Army Corps of Engineers 1992, 1995b). An overview of random variables in a geotechnical context has also been provided by Gilbert (1996).

It should be noted that the selection of any probability distribution (e.g. the lognormal) to characterize a random variable (e.g., the factor of safety) is essentially an *assumption*, made because certain distributions facilitate computations. It cannot in general be proved that a random variable fits a certain distribution, although the *goodness of fit* between a data set and one or more candidate distributions can be assessed by some standard statistical tests, such as the Chi-squared and Kolmogorv-Smirnov tests, found in most statistical texts.

The Lognormal Distribution. The lognormal distribution is of particular interest in geotechnical reliability analysis, as it has certain properties similar to that of some commonly encountered random variables:

- It is a continuous distribution with a zero lower bound and an infinite upper bound.

- As the log of the value is normally distributed, rather than the value itself, it provides a convenient model for random variables with relatively large coefficients of variation (>30%) for which an assumption of normality would imply a significant probability of negative values.

Some random variables often assumed to be lognormally distributed include the coefficient of permeability, the undrained strength of clay, and the factor of safety. The details for making the required transformations to fit lognormal distributions are given in recent Corps' geotechnical guidance (U.S. Army Corps of Engineers 1995b), taken from Shannon and Wilson, Inc., and Wolff (1994).

Moments of Random Variables. When calculating the reliability index or probability of failure by first-order second-moment methods, only the *moments* of a random variable are required; the exact distribution is not required. The first moment about the origin is the *mean* or *expected value*; the second central moment is the *variance*. (Central moments are calculated with respect to the mean). The square root of the variance is the *standard deviation*, and the ratio of the standard deviation to the expected value is the *coefficient of variation*. Calculation of moments is discussed in the references previously cited.

Fitting Distributions and Moments to Test Data. In geotechnical engineering problems, a limited amount of test data is often available to help estimate the moments of parameters of interest (typically strength or permeability). Using standard statistical techniques, the mean and standard deviation of a set of test results can be used to estimate the mean (or expected value) and standard deviation of the random variable.

The sample mean is an *unbiased estimator* of the true or *population mean*. Hence the best estimate of a parameter mean is always the mean of a representative data set. However, with equal likelihood, the sample mean may be greater or less than the true mean, which is unknown. The mean value measured from a randomly selected data set is normally distributed about the true mean with a standard deviation equal in magnitude to the *standard error of the mean*. This error decreases in proportion to the square root of the sample size.

The *standard deviation of the sample values* is a *biased estimator* of *the population standard deviation*. As the uncertainty or variability of values in a large or infinite population is generally greater than that which is measured in a finite sample, estimating the population standard deviation requires increasing the sample standard deviation by an amount which decreases with the square root of the sample size. In other words, the uncertainty in the value of a property at a random point is somewhat greater than the standard deviation calculated from a finite number of tests. Sampling and parameter estimation are further discussed by Harr (1987) and other statistical tests.

Once the moments of the random variable have been estimated as described above, what one actually has is a measure of the uncertainty in the value that would be measured if another sample were tested from a random point in the soil. This value may be referred to as the *point value*. For example, cohesion of clay samples may be measured to estimate the mean and standard deviation of cohesion, which represents the cohesion value at a random point within the same deposit. However, the uncertainty measure required in a seepage or slope stability analysis is typically not the uncertainty in the value at a random point, but rather the uncertainty in the average value over some length. This requires that the variance be further adjusted as discussed later under the heading *spatial correlation.* Accounting for spatial correlation generally leads to some reduction in variance. Hence, estimating an appropriate variance to use in a probabilistic analysis, starting from lab or in situ test values, involves a two-step correction procedure:

- Increasing the sample variance, to obtain the point variance.

- Decreasing the point variance, to obtain the variance of the spatially averaged value required in the analysis.

Some examples of estimating moments for geotechnical parameters of interest to Corps studies are given in Wolff and Wang (1992); Shannon and Wilson, Inc., and Wolff (1994); Wolff (1994); and Wolff et al. (1995). However, these examples do not all include adjustments for spatial correlation effects.

Once the mean and standard deviation of the random variable (either the point value or the spatially averaged value) have been estimated, and perhaps some other assumptions are made, a distribution function (e.g., normal or lognormal) can be assumed if desired and the distribution on the point value can be plotted and visualized.

Typical Coefficients of Variation. Where site-specific data are not available to estimate parameters of random variables, uncertainty can be characterized by assuming that the coefficient of variation of a parameter is similar in magnitude to that observed at other sites. Typical values of coefficients of variation for soil properties have been compiled and reported by Harr (1987). Some example values for parameters involved in stability analysis of gravity monoliths are given by the U.S. Army Corps of Engineers (1993). Compilations for soil strength, permeability, and other parameters of interest to Corps' studies are given in Shannon and Wilson, Inc., and Wolff (1994), and Wolff et al. (1995). Some recent compilations by others include one for soil properties by Lacasse and Nadim (1996) and one for in situ test results by Kulhawy and Trautman (1996).

However, care must be taken when using such typical values, as coefficients of variation alone do not define the *correlation structure* of soil properties, which are defined over a continuum and are spatially correlated. This is further described later in this report under the heading *spatial correlation*.

Independent and Correlated Random Variables. Independent random variables are those for which the likelihood of the random variable assuming a specific value does not depend on the value of any other variable. Where the value of a random variable depends on the value of another random variable, the two are said to be *correlated*. Some examples of random variables that may be correlated are:

- Unit weight and friction angle of sand.

- Preconsolidation pressure and undrained strength of clay.

- The c and ϕ parameters in a consolidated-undrained strength envelope.

Where random variables are correlated, their probability distributions form a joint distribution, and one additional moment, the *covariance*, is necessary to model the parameters when using second-moment methods. An alternative way to express the interdependence is with the correlation coefficient, which relates the covariance to the variances of the two variables.

Calculation of correlation coefficients is further discussed by Tang (1996), U.S. Army Corps of Engineers (1992, 1995a, 1995b), and standard statistical texts. Some investigation into the values of the correlation coefficient between the c and ϕ parameters for various soil materials is reported by Wolff (1985), Wolff and Wang (1992), and Wolff et al. (1995). However, the results are not so consistent as to permit the recommendation of typical values that could be assumed without statistical analysis on specific data.

The effect of parameter correlation is to increase or decrease the total uncertainty, depending on whether correlation is positive or negative. Although parameter correlation can be shown to significantly affect the results of probabilistic analysis, independence of random variables is often assumed in probabilistic analysis. This may be done for two reasons, both computational simplicity and the fact that data are often insufficient to make reliable estimates of the required correlation coefficients.

Spatial Correlation. Random variables that vary continuously over a space or time domain are referred to as *random fields*. In a random field, the variable exhibits *autocorrelation*, the tendency for values of the variable at one point to be correlated to values at nearby points. For example, if one measures the value of soil strength at some point, the uncertainty in the value at a nearby point (say a few feet away), becomes less uncertain, as it is highly correlated to the value of the first point. On the other hand, values measured at considerable distances, say a few hundred feet, may be essentially independent. To characterize a random field, the mean and standard deviation (or variance) are required, plus some quantification of the *correlation stucture.* The correlation structure typically is defined by a *correlation function*, which models the reduction in autocorrelation with distance, and a *characteristic length* or *correlation distance*, a parameter which scales the correlation function.

A classic paper introducing spatial correlation concepts to the geotechnical profession was published by Vanmarcke (1977a). Some recent papers further summarizing the concept include those by DeGroot (1996), Fenton (1996), Lacasse and Nadim (1996), and Phoon and Kulhawy (1996). Some aspects of applying spatial correlation theory have been summarized in a set of simple examples prepared by Wolff (1996c) for the St. Louis District.

To date, spatial correlation concepts have generally not been used in Corps' studies. The methodology in Corps' guidance (U.S. Army Corps of Engineers 1992, 1995b), as well as the related research previously quoted, considers only the expected value and coefficients of variation of random variables and neglects their spatial correlation structure. This has been due to several factors:

- The Corps' methodology has its origin in structural engineering applications, where coefficients of variation alone are sufficient to model uncertainty from one member or component to another (i.e., media are not continuous and the correlation structure need not be quantified).

- The Corps needed to rapidly implement a practical methodology, easily understood and applied by practitioners; consideration of more advanced techniques was deferred pending additional research.

- Methodology needed only to be sufficient to make reasonable comparisons of reliability rather than calculate accurate values.

As the effect of introducing spatial correlation methodology is generally to reduce variances, it could be said that it is consistent and conservative but technically incorrect to perform probabilistic analysis without considering spatial correlation.

First-Order Second-Moment Reliability Methods

The primary approach in Corps guidance to date (e.g., U.S. Army Corps of Engineers 1992; 1993, 1995b) has been the use of FOSM methods. In this approach, the same as the basis for structural design codes, uncertainty in performance is taken to be a function of uncertainty in model parameters or in the model itself. The expected values and standard deviations of the random variables (and sometimes model accuracy) are used to estimate the expected value and standard deviation of a performance function, such as the factor of safety against slope instability.

The Reliability Index. The usual output of FOSM methods is the *reliability index*, β. Given some performance function and limit state, the reliability index is the number of standard deviations of the performance function by which the expected value of the performance function exceeds the limit state. The concepts of FOSM methods and the reliability index are illustrated in Figure 2.

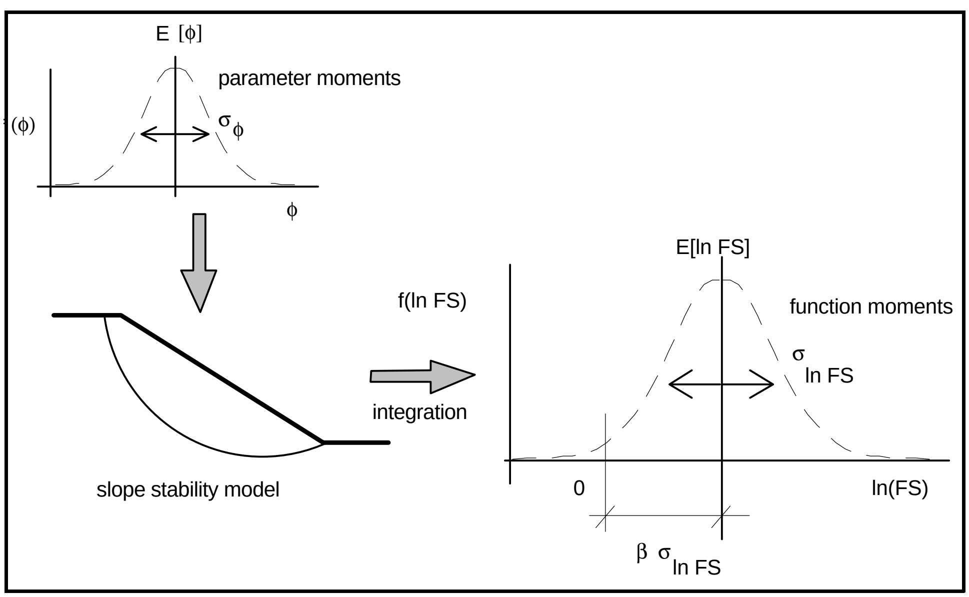

Figure 2. Method of moments -- reliability index approach (after Wolff (1996a))

The reliability index provides a measure of *relative or comparative reliability* without having to assume a probability distribution for the performance function. A complete distribution would be required to calculate the probability of failure, but its form is generally unknown. The reliability index concept was popularized in structural code development, to enable design of structural members to desired levels of relative reliability, without knowing or having to assume probability distributions for the performance functions. The concept of relative reliability is supported in early Corps guidance (U.S. Army Corps of Engineers 1992), which states that the reliability index values are "sufficiently accurate to rank the relative reliability of various structures and components, but they are not absolute measures of reliability." The same ETL suggested that "Target reliability indices may be established for critical lock and dam components and performance modes."

A step-by-step description of FOSM methodology, working from random variables through to β, is given in Corps guidance (U.S. Army Corps of Engineers 1992, 1993, 1995b) and related research reports (Wolff and Wang 1992; Wolff 1994; Shannon and Wilson, Inc., and Wolff 1994).

Probability of Failure or Unsatisfactory Performance. Although comparative β values would be sufficient to rank structures for repair, and target β values would provide decision strategy regarding what to repair, the Corps' economic analysis methodology requires probability values to permit full development of an event tree and probabilistic modeling of economic consequences of unsatisfactory performance. In probabilistic literature, the probability that the performance function is more adverse than the limit state is termed the *probability of failure* Pr(f). However, some Corps guidance uses the term *probability of unsatisfactory performance* Pr(U) to recognize the fact that the event under consideration may not be catastrophic. To obtain Pr(f) or Pr(U) from β, a probability distribution on the performance function must be assumed. A normal distribution is generally used for ease of calculation; however, the performance function is often then taken as ln FS (or ln capacity/demand), implying that the factor of safety is lognormally distributed. Given this assumption and the value of β, the required probability values are easily calculated from the properties of the assumed distribution.

Taylor's Series Mean Value Method. To calculate β**,** the moments of the performance function must be calculated from the moments of the parameters. The most common method used in Corps practice is the Taylor's series method, based on a Taylor's series expansion of the performance function about the expected values. The expected value of the performance function is obtained by evaluating the function using the expected values of the parameters. The variance is obtained by summing the products of the partial derivatives of the performance function (taken at the mean parameter values) and the variances of the corresponding parameters. The detailed equations are given in Corps guidance (U.S. Army Corps of Engineers 1992, 1995b), Wolff and Wang (1992, 1993), Shannon and Wilson, Inc., and Wolff (1994), and Wolff et al. (1996).

In Corps practice, the required partial derivatives are calculated numerically using an increment of plus and minus one standard deviation, centered on the expected value. This specific increment is unique to the Corps (numerical derivatives are often calculated using very small increments), and was chosen to capture some of the behavior of nonlinear functions even though the Taylor's series method is exact only for linear functions. (For a linear function, any increment will yield the same results). It also leads to computational simplicity.

Point Estimate Method. An alternative method to the Taylor's series method is the point estimate method, developed by Rosenblueth (1975, 1981), and summarized by Harr

(1987). It is also discussed more briefly in Corps guidance (U.S. Army Corps of Engineers 1992, 1995b) and the related reference previously cited. In the point estimate method, no calculations are made at the mean value, but rather the moments of the performance function are determined by evaluating it at a set of combinations of high and low parameter values, with the results weighted by factors. The point estimate method has been less popular in practice because it requires more evaluations of the performance function when the number of random variables exceeds two. However, it may better capture the behavior of nonlinear functions. Some detailed comparisons of the two methods for a number of real problems are given by Wolff and Wang (1992, 1993), Wolff et al. (1996), and Wolff (1996a).

Hasofer - Lind Method. A potential problem with both the Taylor's series method and the point estimate method is their lack of invariance for nonlinear performance functions. If a performance function and limit state can be expressed in more than one equivalent way (e.g., Capacity / Demand = 1 or Capacity - Demand = 0), these two functions will yield different values for the reliability index. Related problems are computational difficulties in determining derivatives of very nonlinear functions such as bearing capacity. For example, an example analysis in U.S. Army Corps of Engineers (1993) uses only the mean values of rock strength parameters to circumvent this difficulty.

A more general definition of the reliability index, which is invariant and reduces to the mean-value definition for linear functions, was developed by Hasofer and Lind (1974). In their method, the Taylor's series is expanded, not about the mean or expected value, but about an unknown point termed the *failure point*. An iterative solution is required. Examples of the methodology are given by Ang and Tang (1985). Many published analyses of geotechnical problems have not used the Hasofer-Lind method, probably due to its complexity, especially for implicit functions such as those in slope stability analysis. The use of the mean-value Taylor's series method or the point estimate method, and neglect of the invariance problem, introduces error of an unknown magnitude in probabilistic analyses. The degree of error depends on the degree of nonlinearity in the performance function and the coefficients of variation of the random variables.

Monte Carlo Simulation

An alternative means to estimate the expected value and standard deviation of the performance function is the use of *simulation methods,* often referred to as *Monte Carlo methods or Monte Carlo simulation*. In Monte Carlo simulation, values of the random variables are generated in a fashion consistent with their probability distribution, and the performance function is calculated for each generated set. The process is repeated numerous times, typically thousands, and the expected value, standard deviation, and probability distribution of the performance function are taken to match that of the calculated values. **Advantages** of the Monte Carlo method include the following:

- It permits one to estimate the shape of the distribution on the performance function, permitting more accurate estimation of probability values (however, see disadvantages below).

- For explicit performance functions, it is easily programmed with simulation software such as the Excel© add-in @RISK©.

Disadvantages include the following:

- The shapes of the distributions on the random variables must be known or assumed; hence the distribution obtained for the performance function is only accurate to the extent that these are accurate.

- Accuracy of the estimated values is proportional to the square root of the number of iterations; hence doubling the accuracy requires increasing the number of iterations fourfold.

- Implicit functions requiring special programs (such as slope stability analysis) require additional special programming for Monte Carlo analysis.

Despite these disadvantages, Monte Carlo analysis is likely to become increasingly common in lieu of FOSM methods as computing capabilities continue to improve.

Some Comments on the Use and Meaning of β or Pr(u)

Potential for Overlooking Some Performance Modes. A shortcoming of using only FOSM or Monte Carlo methods in reliability analysis is the potential for overlooking some performance modes. Christian (1996) notes that

> *The analyses leading to computed values of β and p_f can include contributions from only those factors that the analyst has recognized and incorporated into the calculations. If the analyst has ignored some important factor, its contribution to the probability of failure will also be ignored, and the computed value of p_f will be correspondingly too low. A great many slope failures have been found to be due to features that were overlooked by the designers, or unanticipated factors introduced during construction.*

As FOSM or Monte Carlo methods require characterization of random variables and selection of performance functions, emphasis may be given to those modes for which this is easily done. The careful preparation of an event tree by a multidisciplinary team as the first step in a risk analysis may alleviate this problem as it promotes consideration of all possible unsatisfactory performance events, whether or not they are easily modeled by random variables.

Physical Meaning of Probability of Failure for Existing Structures. The probability of failure (or unsatisfactory performance) value for an existing structure presents something of a philosophical paradox. As it is a transformation of the uncertainty in parameter values to uncertainty in performance, its meaning for new structures could be construed as follows:

> *Given that there is the specified uncertainty in parameter values before construction, what is the probability that the value of the performance function for the as-constructed structure will be to the adverse side of the limit state?*

Hence, the probability values from an FOSM analysis are implied to have a “per structure” frequency. A probability of failure of 1 in 1000 could be construed to mean that, given 1000 similar structures constructed under independent, but statistically replicate conditions, one failure would be expected upon first loading of the modeled condition.

For a still-existing structure that has been subjected to a modeled load, it can obviously be observed that the structure has not failed. Nevertheless, a probability of failure value can be associated with that event. Hence, the probability of failure calculated for an existing structure should be construed not as a contradiction of fact, but as a comparative measure of reliability, suitable for judging the reliability of the structure and considered performance mode relative to other structures or modes.

Lack of Time Dimension in FOSM Methods. It must be reemphasized that FOSM methods and β provide a measure of reliability with respect to a load event, but provide no intrinsic information regarding lifetimes or time-based probabilities of failure or unsatisfactory performance (U.S. Army Corps of Engineers 1995b). To achieve a time-based reliability analysis, some other random variable must have a time basis, such as the load event considered (probability of occurrence per year), pool level or earthquake acceleration (probability of occurrence per year), or some time-random event (occurrence of scour or initiation of a seep). FOSM methods can then be used to develop conditional probabilities to follow the time-based antecedent event in the event tree.

Frequency-based Reliability Methods

In some circumstances, notably where data on actual lifetimes of components are more accurate, more available and better understood than parameter uncertainty and performance functions, and where it is desired to construct hazard functions, frequency-based reliability methods may be employed to advantage. This is the most common approach used in designing mechanical, electrical, and electronic parts, for which it is fairly easy to construct a number of replicate specimens and test them to failure. Such an observational approach permits direct verification of the distribution of lifetimes without resort to inferring them from more indirect approaches. For large civil engineering structures, testing replicate specimens to failure is often out of the question, as structures are unique and expensive.

A detailed treatment of lifetime distributions is provided by Lewis (1996), Nelson (1982), and others. The methodology has been developed to considerable levels of sophistication, although much is built on the Weibull distribution, which permits time-varying hazard functions, and for which the exponential lifetime distribution of a Poisson process is a special case.

The modeling of event frequency using the Weibull distribution fit to observed events was reviewed in the methodology report prepared for the St. Louis District by Shannon and Wilson, Inc., and Wolff (1994) and some examples were provided. An extended review of the methodology for certain special cases was prepared by Wolff (1996b). These techniques were used for certain aspects of the Upper Mississippi River study to develop hazard functions for performance modes for which FOSM techniques are not easily applied. They were also used to model the random occurrence of seepage incidents in the Walter F. George dam study (U.S. Army Engineer District, Mobile 1997).

Subjectively Determined Probability Values

For some probability values required in an event tree, there may be neither sufficient information (parameter variability and performance function) to employ FOSM methods nor sufficient reliable historical data of similar events to employ frequency-based methods. If it is necessary to develop conditional probability values for an event tree under these circumstances, a final option is to estimate the values based solely on engineering judgment. Although this may appear tantamount to guessing, there are established ways to structure the estimation of such values by a panel of experts, moved toward a consensus in an interactive and iterative exercise involving information sharing and feedback.

Although the use of expert elicitation in Corps' studies has been limited (e.g. U.S. Army Engineer District, Mobile 1997), some other agencies and entities owning dams have used it more commonly than FOSM methods and β values. The application of expert elicitation to dam safety, with some reference to the methods and problems of establishing subjective probability values has been discussed by Vick and Stewart (1996), who draw on more general research on judgmental probability assessment by those in the behavioral sciences. They state known problems with the process, such as overconfidence bias, motivational bias, and problems with cognitive discrimination among extremely low probability values. Both Vick and Stewart (1996) and VonThun (1996) provide case histories of such analyses, the former for Canadian hydropower projects and the latter for a U.S. Bureau of Reclamation project.

System Reliability

In some cases, it is necessary to establish the reliability of a system given the reliability of its components. Solutions for simple parallel and series systems are given in Corps guidance (U.S. Army Corps of Engineers 1992, 1995b). Solutions for more complex systems can sometimes be obtained by reducing the system to combinations of series and parallel systems. For some cases of complex and redundant systems, only bounds on the reliability values can be obtained. System reliability is discussed in more detail in many of the standard references cited.

For comparative economic analysis for Corps' investment decisions, the issue of complex systems has been approached by noting that the reliability of a few critical components often governs the system. Hence, an analysis of such identified components has generally been used as the basis for reliability analysis.

An example of simple systems reliability is given by Wolff (1994) for flood control levees. In that report, it is assumed that the total probability of failure for a levee exposed to a number of risks can be modeled assuming that the performance modes form an independent series system.

Special Issues in Geotechnical Engineering

Some Unique Aspects in Geotechnical Problems. Some geotechnical engineering problems have a number of unique aspects. These aspects include the following:

- In geotechnical engineering, coefficients of variation are related to the **variability of natural materials**, which may need to be assessed on a site-specific basis.

- Geotechnical parameters may have relatively **high coefficients of variation** (the value for the coefficient of permeability may exceed 100 percent) and may be correlated (e.g., c and ϕ).

- Soil strength parameters can be defined and analyses performed in either a **total stress** context or an **effective stress** context. In the former, the uncertainty in strength and pore pressure are lumped; in the latter, they are treated separately.

- Soils are continuous media where properties vary from point to point, requiring consideration of **spatial correlation**.

- For problems such as slope stability, the **location of the critical free body must be searched out**. Furthermore, its location varies with parameter values, and varying parameter values (in an FOSM or Monte Carlo analysis) results in different free-body locations for each set of parameter values.

- Although one slip surface may be "critical," a slope can fail on any of an infinite number of slip surfaces; hence **a slope is a system** of possible failure surfaces which are correlated to some extent.

- Some **earth structures** such as levees may be **exceedingly long**, such as levees which may be tens of miles long. These can be treated as a number of equivalent independent structures; however, determining the appropriate length and number is problematical, and the reliability of the system may be sensitive to the assumptions made.

Complexities such as those cited above have slowed the adoption of probabilistic methods in geotechnical engineering, both within and outside the Corps.

Strength Parameters from Triaxial Tests. The parameters c and ϕ measured from triaxial tests are not measured uniquely on single samples, but are interpreted from the results of several tests on replicate samples tested at different confining pressures. Hence, the determination of probabilistic moments on c and ϕ from test data is not straightforward. Ur-Rasul (1995) considered eleven methods to do so. These are summarized with recommendations by Wolff et al. (1995) and are briefly discussed in Wolff (1996a).

Free-body and Critical Slip Surface Issues in Slope Stability Analysis. In slope stability analysis, a large number of free bodies are systematically considered until a critical free body is found which minimizes the factor of safety. This critical deterministic surface may not coincide with the critical probabilistic surface. At least three approaches can and have been considered in assigning a reliability index to a slope:

a. Take the reliability index as that for the critical deterministic surface.

b. For each combination of strength parameters considered in an FOSM or Monte Carlo analysis, search the critical slip surface and use the factors of safety for this set of mixed surfaces to calculate β.

c. Generate candidate slip surfaces, calculate β for each (varying strength parameters while holding the surface geometry fixed), and systematically search for the surface of minimum β.

The first approach above will not, in general, provide a reasonable indication of the reliability of a slope, as there may be other surfaces which give lower β values.

The second approach, sometimes referred to as a *floating surface,* as β is calculated from results from a number of different surfaces, has been used in several studies, including Appendix B to the ETL transmitting this Appendix (Wolff 1994), and Shannon and Wilson, Inc., and Wolff (1994) as it is computationally convenient (results of UTEXAS3 analyses for different strength inputs can be used directly to calculate β) and was considered to provide a measure of the reliability for the entire slope as a system. However, it raises a philosophical issue regarding its meaning as the resulting β value is not associated with any single free body. As programs become available for the third approach, it is recommended that it be followed. In the meantime sufficient surfaces should be analyzed to ensure that the surface of minimum reliability index has been located as well as practicable.

Limited research on the third approach, published by Wolff et al. (1995) and further investigated by Hassan (1996), indicates that calculating β for surfaces of fixed geometry and systematically searching for a fixed surface of minimum β may locate surfaces with significantly lower β values than the preceding approaches.

Where the third approach is followed, the reliability index of a slope is commonly taken as the value corresponding to the slip surface of minimum β. However, a slope is a system comprised of an infinite number of possible slip surfaces, each of which can fail, and each with different β. The resulting system is analogous to a large truss, which would have a system reliability index lower than that of its critical member. The problem is further complicated because closely spaced slip surfaces are highly correlated. The slip surface of minimum β is in fact a lower bound on the β value for the slope, which is not easily determined.

Application of Spatial Correlation Theory to Slope Stability and Seepage Analysis. As previously noted, soils are random fields (continuous media with spatially correlated values). Where the correlation distance is shorter than the scale of the free body or cross section analyzed in a stability or seepage analysis, parameter variances must be reduced to represent the uncertainty in the average property over the considered cross section. A more refined approach is to consider that individual slices in a stability analysis or individual finite elements in a seepage analysis each have random parameter values that are correlated with those of adjacent slices or elements. The required correlation coefficients are related to geometric size of the elements and correlation structure of the media. An introduction to spatial correlation issues is provided by Vanmarcke (1977a, 1977b). A summary and examples with additional references were provided for the St. Louis District by Wolff (1996c). Neglecting spatial correlation, as is commonly the case for Corps' studies, implicitly assumes that the correlation distance is larger in dimension than the considered section.

Application of Spatial Correlation Theory to Long Earth Structures. A second consideration of spatial correlation is the natural variability of soil properties in the direction normal to the two-dimensional cross section analyzed. A slope stability or seepage analysis made on a two-dimensional section is assumed representative of some unspecified length of embankment. However, a 1-mile length of levee or embankment, even on very uniform materials, is less reliable than a 100-ft length of the same embankment. To calculate the reliability of a long embankment as a series system, analogous to a chain of independent links, a long section must be converted to a number of statistically equivalent independent sections. This in turn may require more detailed knowledge of the correlation structure than is generally available. The problem of slope failures in long embankments has been considered by Vanmarcke (1977b). Vrouwenvelder (1987) uses an upper and lower bound system reliability approach and a correlation length of 500 m in an analysis of Dutch levee systems.

Examples of Probabilistic Analysis

A few examples of case histories of geotechnical probabilistic analyses and research studies are briefly reviewed below to provide the reader a sense of the development of the methodology and refer the reader to more detailed examples.

Wappapello Dam, St. Louis District. Wolff et al. (1988) reported an analysis by the St. Louis District of the probability of an earthquake-induced pool release at Wappapello

Dam in southeastern Missouri. Although the dam is in a seismic area, it also has a relatively high normal freeboard. The assessment combined the probability of foundation liquefaction integrated over a range of possible earthquake magnitudes, the probability of sliding given liquefaction, the probability distribution on slide scarp elevation given sliding, and the probability of overtopping given slide scarp elevation and pool level.

Shelbyville Dam, St. Louis District. Wolff (1991) reported the results of comparative probabilistic slope stability analysis for conditions before and after repair of a slide at Shelbyville Dam. Using the point estimate method, it was demonstrated that placement of a rock berm significantly reduced the probability of failure.

Research on Navigation Structures for Guidance Development. Wolff and Wang (1992, 1993) published a set of example analyses and methodology comparisons based on navigation structures on the Monongahela and Tennessee-Tombigbee river systems. These included probabilistic characterization of soil and rock strength, comparison of the Taylor's series and point estimate methods to calculate β, and an evaluation of the improvement in reliability achieved by remedial action.

A later report by Shannon and Wilson, Inc., and Wolff (1994) for the St. Louis District provided a series of examples for sliding and overturning analysis for gravity structures, slope stability analysis, and various types of seepage analysis. In addition to providing examples for calculating β values, the report illustrated the fitting of Weibull distribution to historical events to obtain hazard functions.

Research on Levees for Guidance Development. Wolff (1994) provided a set of examples for levee reliability analysis considering a variety of failure modes. Conditional probability-of-unsatisfactory-performance functions were developed as functions of floodwater elevations, and the resulting functions were combined assuming the various modes form a simple series system. The complete report accompanies this ETL as Appendix B.

Hodges Village Dam, New England Division. A probabilistic assessment of seepage problems at the Hodges Village Dam was prepared by the New England Division (U.S. Army Engineer Division, New England 1995). Hodges Village Dam is a normally dry flood control dam built on very pervious sands and gravels without a cutoff. Residential development is present adjacent to the toe of the dam. During past high-water events, extensive seepage with damaging erosion has occurred, and the potential for a safety problem at higher water levels was of concern. Although the nature of the problem permitted a decision to remediate without a probabilistic assessment, one was performed in support of the economic studies. Similar to the approach outlined in the levee research described in the preceding paragraph, a stage-exceedance probability function was used to develop probability values for annual high pool elevations. The conditional probability of exit gradients in excess of critical values, given pool level, was calculated using probabilistic seepage analyses. The random variables in the analyses were the permeability ratios of subsurface strata. A range of permeability ratios was determined within which the seepage model could be calibrated to match past events; a probability distribution on the "true permeability ratio" was fit to span that range.

Walter F. George Dam, Mobile District. A second risk analysis involving seepage problems was performed for the Walter F. George Dam by the Mobile District (U.S. Army Engineer District, Mobile 1997). Unlike the Hodges Village Dam, for which a finite-element analysis could be performed to calculate gradients in pervious soils, seepage through the foundation at the Walter F. George project occurs in solutioned limestone, and uncontrolled seepage events have occurred at seemingly random locations on random occasions unrelated to pool level. These events have been repaired by exploring the lake area for seepage inlets and plugging them with concrete or grout. Having no situation readily amenable to analytical modeling, the risk assessment was performed using a combination of frequency-based reliability methods fit to historical events and subjectively determined probability values based on expert elicitation. Given the set of historical events, annual probabilities of new events were taken to be increasing with time due to the continued solutioning of the limestone. The expert panel estimated probability values for future seeps occurring at various locations, for locating the source of the seep in sufficient time, for being able to repair the seeps given that they are located, and for various structural consequences of uncontrolled seepage.

Summary

Probabilistic methods are being used by the Corps of Engineers for risk analysis in support of economic planning studies for project rehabilitation, and are being considered for other applications. The framework of such risk analysis is an event tree, a pictorial representation of a system of possible events and outcomes connected by conditional probability values. The required probability values can be obtained by three approaches. The first of these, based on parameter uncertainty and performance functions, has been the most widely used to date. The second, based on fitting probability distributions to historical events, has some advantages where knowledge of such events is more complete than knowledge of parameter uncertainty and performance functions. The third approach, subjective estimation of probability values by expert elicitation, has had only limited application in the Corps, but has been used by some other agencies. Corps guidance, other publications providing details of all three of these approaches, and example case histories have been reviewed and a number of references have been provided to give the reader a broad perspective on the state of risk analysis in geotechnical engineering.

Current Corps' guidance for probabilistic analysis has a good experience record given the short time frame it has been used and the rapid rate at which it was put into practice. However, geotechnical engineering problems have a number of unique aspects not yet fully treated in such guidance. Many of these center on the fact that soils and rock are continuous media rather than discrete members, and the fact that soils and rock are natural materials rather than constructed or manufactured materials. Notable among these are characterization of strength parameters, spatial correlation considerations, and system reliability of slopes. Additional refinements to the methodology will need to be developed in the future as the need to perform risk analyses of geotechnical problems continues and experience with the techniques is gained.

References

Ang, A. H.-S., and Tang, W. H. (1975), *Probability Concepts in Engineering Planning and Design, Volume I: Basic Principles*, John Wiley and Sons, New York.

Ang, A. H.-S., and Tang, W. H. (1985), *Probability Concepts in Engineering Planning and Design, Volume II: Decision, Risk and Reliability*, John Wiley and Sons, New York.

Benjamin, J., and Cornell, C. A. (1970), *Probability, Statistics and Decision for Civil Engineers*, McGraw-Hill, New York.

Christian, J.T. (1996), "Reliability Methods for Stability of Existing Slopes," *in Uncertainty in the Geologic Environment: From Theory to Practice*, Proceedings of

Uncertainty '96, ASCE Geotechnical Special Publication No. 58, C.D. Shackelford, P.P Nelson, and M.J.S. Roth, eds., pp. 409-418.

Christian, J. T., Ladd, C. C., and Baecher, G. B. (1992), "Reliability and Probability in Stability Analysis," *Proceedings of Specialty Conference on Stability and Performance of Slopes and Embankments - II*, ASCE, Vol. 2, No. 12, pp. 1071-1111.

Christian, J. T., Ladd, C. C., and Baecher, G. B. (1994), "Reliability Applied to Slope Stability Analysis," *Journal of Geotechnical Engineering*, ASCE, Vol. 120, No. 12, pp. 2180-2207.

DeGroot, D.J. (1996), "Analyzing Spatial Variability of In-situ Soil Properties," *in Uncertainty in the Geologic Environment: From Theory to Practice*, Proceedings of Uncertainty '96, ASCE Geotechnical Special Publication No. 58, C.D. Shackelford, P.P Nelson, and M.J.S. Roth, eds., pp. 210-238.

Ellingwood, B., Galambos, T. V., MacGregor, J., and Cornell, C. A. (1980), "Development of a Probability-Based Load Criterion for National Bureau of Standards A58," *NBS Special Publication 577*, National Bureau of Standards, Washington, DC.

Fenton, G. A. (1996), "Data Analysis / Geostatistics," Chapter 4 *of Probabilistic Methods in Geotechnical Engineering*, notes from workshop presented at ASCE Uncertainty '96 Conference, Madison, WI, July 31, 1996, sponsored by ASCE Geotechncial Safety and Reliability Committee, G. A. Fenton, ed.

Frudenthal (1947), "The Safety of Structures," *ASCE Transactions*, Vol. 112, 1947, pp. 125-159.

Gilbert, R. B. (1996), "Basic Random Variables," Chapter 2 *of Probabilistic Methods in Geotechnical Engineering*, notes from workshop presented at ASCE Uncertainty '96 Conference, Madison, WI, July 31, 1996, sponsored by ASCE Geotechnical Safety and Reliability Committee, G. A. Fenton, ed.

Hahn, G. J., and Shapiro, S. S. (1967), *Statistical Models in Engineering*, John Wiley and Sons, New York.

Harr, M. E. (1987), *Reliability-Based Design in Civil Engineering*, McGraw-Hill, New York.

Hassan, A. (1996). Personal Communication.

Hasofer, A. A. and Lind, A. M. (1974), "An Exact and Invariant Second-Moment Code Format," *Journal of the Engineering Mechanics Division*, ASCE, Vol. 100, pp. 111-121.

Kulhawy, F., and Trautman, C. H., (1996), "Estimation of In-Situ Test Uncertainty," in *Uncertainty in the Geologic Environment: From Theory to Practice*, Proceedings of Uncertainty '96, ASCE Geotechnical Special Publication No. 58, C.D. Shackelford, P.P Nelson, and M.J.S. Roth, eds., pp. 269-286.

Lacasse, S. and Nadim, F. (1996), "Uncertainties in Characterizing Soil Properties," in *Uncertainty in the Geologic Environment: From Theory to Practice*, proceedings of Uncertainty '96, ASCE Geotechnical Special publication No. 58, C.D. Shackelford, P.P Nelson, and M.J.S. Roth, eds., pp. 49-75.

Lewis, E. E. (1996), *Introduction to Reliability Engineering*, John Wiley and Sons, New York.

Moses, F. and Verma, D. (1987), "Load Capacity Evaluation of Existing Bridges," *National Cooperative Highway Research Program Report 310*, Transportation Research Board, National Research Council, Washington, DC.

National Research Council (1995), *Probabilistic Methods in Geotechnical Engineering*,

Nelson, W. (1982), *Applied Life Data Analysis*, John Wiley and Sons, NY.

Phoon, K. K. and Kulhawy, F. H. (1996), "On Quantifying Inherent Soil Variability," in *Uncertainty in the Geologic Environment: From Theory to Practice*, Proceedings of Uncertainty '96, ASCE Geotechnical Special Publication No. 58, C.D. Shackelford, P.P Nelson, and M.J.S. Roth, eds., pp.326-240.

Rosenblueth, E. (1975), "Point Estimates for Probability Moments," *Proceedings of the National Academy of Science*, USA, 72(10).

Rosenblueth, E. (1981), "Two-Point Estimates in Probabilities," *Applied Mathematical Modeling*, 5.

Shannon and Wilson, Inc., and Wolff, T. F. (1994), "Probability Models for Geotechnical Aspects of Navigation Structures," report to the St. Louis District, U. S. Army Corps of Engineers.

Tang, W.H. C. (1996), "Correlation, Multiple RV's and System Reliability," Chapter 3 *of Probabilistic Methods in Geotechnical Engineering*, notes from workshop presented at ASCE Uncertainty '96 Conference, Madison, WI, July 31, 1996, sponsored by ASCE Geotechncial Safety and Reliability Committee, G. A. Fenton, ed.

U.S. Army Corps of Engineers (1992), "Reliability Assessment of Navigation Structures," *ETL 1110-2-532*, 1 May 1992.

U.S. Army Corps of Engineers (1993), "Reliability Assessment of Navigation Structures: Stability Assessment of Gravity Structures," *ETL 1110-2-321*, 31 December 1995.

U.S. Army Corps of Engineers (1995a), "Reliability Assessment of Pile-Founded Navigation Structures," *ETL 1110-2-354*, 31 August 1995.

U.S. Army Corps of Engineers (1995b), "Introduction to Probability and Reliability Methods for Use in Geotechnical Engineering," *ETL 1110-2-547*, 30 September 1995.

U.S. Army Engineer Division, New England (1995), "Hodges Village Dam, Major Rehabilitation Evaluation Report," June 1995.

U.S. Army Engineer District, Mobile (1997), "Walter F. George Lock and Dam, Major Rehabilitation Evaluation Report, Prevention of Potential Structural Failure," March 1997.

Ur-Rasul, I. (1995), "Characterizing Soil Strength for Probabilistic Analysis Working from Test Results: A Practical Approach," MS thesis, Michigan State University, December 1995.

Vanmarcke, E. H. (1977a), "Probabilistic Modeling of Soil Profiles," *Journal of the Geotechnical Engineering Division*, ASCE, Vol 103, No. GT11, November 1977, pp. 1227-1246.

Vanmarcke, E. H. (1977b), "Reliability of Earth Slopes," *Journal of the Geotechnical Engineering Division*, ASCE, Vol 103, No. GT11, November 1977, pp. 1247-1265.

Vick, S. G., and Stewart, R. A. (1996), "Risk Analysis in Dam Safety Practice," *in Uncertainty in the Geologic Environment: From Theory to Practice*, Proceedings of Uncertainty '96, ASCE Geotechnical Special Publication No. 58, C.D. Shackelford, P.P Nelson, and M.J.S. Roth, eds., pp. 586-601.

VonThun, J. L. (1996), "Risk Assessment of Nambe Falls Dam," in *Uncertainty in the Geologic Environment: From Theory to Practice*, Proceedings of Uncertainty '96, ASCE Geotechnical Special Publication No. 58, C.D. Shackelford, P.P Nelson, and M.J.S. Roth, eds., pp. 604-622.

Vrouwenvelder, A.C.W.M. (1987), "Probabilistic Design of Flood Defenses," Report No. B-87-404. IBBC-TNO (Institute for Building Materials and Structures of the Netherlands Organization of Applied Scientific Research), The Netherlands.

Whitman, R. V. (1996), "Organizing and Evaluating Uncertainty in Geotechnical Engineering," in *Uncertainty in the Geologic Environment: From Theory to Practice*, Proceedings of Uncertainty '96, ASCE Geotechnical Special Publication No. 58, C.D. Shackelford, P.P Nelson, and M.J.S. Roth, eds., pp. 1-28.

Wolff, T. F. (1985), "Analysis and Design of Embankment Dam Slopes: A Probabilistic Approach," thesis submitted to the faculty of Purdue University in partial fulfillment of the requirements for the degree of Doctor of Philosophy.

Wolff, T. F. (1987), "Slope Design for Earth Dams," in *Reliability and Risk Analysis in Civil Engineering 2*, Proceedings of the Fifth International Conference on Applications of Statistics and Probability in Soil and Structural Engineering, Vancouver, BC, Canada, pp. 725-732.

Wolff, T. F. (1991), "Embankment Design Versus Factor of Safety: Before and After Slide Repair," *International Journal of Numerical and Analytical Methods in Geotechnical Engineering*, Vol 15, No. 1, pp. 41-50.

Wolff, T. F. (1994), "Evaluating the Reliability of Existing Levees," research report prepared for U.S. Army Engineer Waterways Experiment Station, Michigan State University, September 1994.

Wolff, T. F. (1995), Probabilistic Methods in Engineering Analysis and Design, notes from short course for the Jacksonville District, U.S. Army Corps of Engineers, March 1995.

Wolff, T. F. (1996a), "Probabilistic Slope Stability in Theory and Practice," *in Uncertainty in the Geologic Environment: From Theory to Practice*, Proceedings of Uncertainty '96, ASCE Geotechnical Special Publication No. 58, C.D. Shackelford, P.P Nelson, and M.J.S. Roth, eds., pp. 419-433.

Wolff, T. F. (1996b), "Application of Time-based Reliability Analysis to Corps of Engineers' Geotechnical Engineering Problems," report prepared for St. Louis District, U.S. Army Corps of Engineers, under subcontract to Shannon and Wilson, Inc., St. Louis, MO, November 1996.

Wolff, T. F. (1996c), "Application of Spatial Averaging and Spatial Correlation to Corps of Engineers' Geotechnical Engineering Problems," report prepared for St. Louis District, U.S. Army Corps of Engineers, under subcontract to Shannon and Wilson, Inc., St. Louis, MO, December 1996.

Wolff, T. F., Demsky, E. C., Schauer, J., and Perry, E. (1996), "Reliability Assessment of Dam and Levee Embankments," in *Uncertainty in the Geologic Environment: From Theory to Practice*, Proceedings of Uncertainty '96, ASCE Geotechnical Special Publication No. 58, C.D. Shackelford, P.P Nelson, and M.J.S. Roth, eds., pp. 636-650.

Wolff, T. F., Hassan, A., Khan, R., Ur-Rasul, I., and Miller, M. (1995), "Geotechnical Reliability of Dam and Levee Embankments, " Report for the U.S. Army Engineer Waterways Experiment Station, by Michigan State University, September 1995.

Wolff, T. F., Hempen, G. L., Dirnberger, M. M., and Moore, B. H. (1988), "Probabilistic Analysis of Earthquake-Induced Pool Release, " *Second International Conference on Case Histories in Geotechnical Engineering*, University of Missouri - Rolla, pp. 787-794.

Wolff, T. F., and Wang, W. (1992), "Engineering Reliability of Navigation Structures," research report, Michigan State University, for U.S. Army Engineer Waterways Experiment Station, Vicksburg, MS.

Wolff, T. F., and Wang, W. (1993), "Reliability Analysis of Navigation Structures," in *Geotechnical Practice in Dam Rehabilitation*, Specialty Conference, Geotechnical Engineering Division, ASCE, North Carolina State University, Raleigh, North Carolina, ASCE Geotechnical Specialty Publication No. 35, pp. 159-173.

Wu, T. H. (1996), "Events, Fault Trees, Bayes' Theorem," Chapter 1 of *Probabilistic Methods in Geotechnical Engineering*, notes from workshop presented at ASCE Uncertainty '96 Confernce, Madison, WI, July 31, 1996, sponsored by ASCE Geotechnical Safety and Reliability Committee, G. A. Fenton, ed.